GROUP DELAY OPTIMIZATION IN CASCADED OPTICAL RING RESSONATOR-BASED OPTICAL BEAMFORMING NETWORKS

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ABSTRAK

Optical Beamforming Networks (OBFN) yang berdasarkan aplikasi *Optical ring resonator* (ORR) sebagai faktor penunda dapat digunakan untuk mengoptimalkan daya sinyal yang diterima oleh pesawat terbang. Hal ini sangat esensial untuk aplikasi komunikasi antara satelit dan pesawat terbang sebagai solusi tersedianya akses internet kecepatan tinggi di dalam kabin pesawat. Sistem ini menggunakan *Phased Array Antenna* (PAA) sebagai antenna penerima sinyal, yang merupakan rangkaian beberapa antena kecil dan datar yang dapat menerima sejumlah sinyal dengan perbedaan waktu penerimaan sinyal masing-masing tergantung dari sudut penerimaannya. Perbedaan waktu penerimaan antar antena-antena kecil tersebut membuat daya sinyal total yang dihasilkan tidak akan maksimal, karena terdapat perbedaan fasa sinyal yang diterima. Artikel ini menyajikan sebuah metode optimasi waktu tunda penetimaan sinyal, sehingga perbedaan fasa antar sinyal dapat diminimalkan, yang pada akhirnya akan membuat daya sinyal yang diterima akan maksimal. Dalam artikel ini, dapat diverifikasi bahwa berdasarkan hasil simulasi yang dilakukan, metode optimasi yang dipaparkan dapat digunakan untuk mengatur *Optical Beamforming Networks* (OBFN) untuk mencapai daya optimal terhadap sudut penerimaan yang bervariasi.

Kata kunci: optical beamforming network, optical ring resonator, phased array antenna, non-linear optimization

ABSTRACT

Cascaded optical ring resonator-based optical beamforming networks can be used to optimize the signal power received by planes. This is essential for airplanes-sattelite communication as a solution for a high-speed internet access inside airplanes cabin. This system relies on phased array antenna (PAA), an array of a very small and flat antenna elements which will receive a time-delayed version of desired signal from specific angle. The delay time will create phase differences between each received signal, and in consequence, lower the signal power. We propose a method to optimize the group delay of the received signals in order to obtain maximum signal power. We simulate the optimization algorithm using a set of data, which consists of full input signals. To analyze the accuracy, the desired signal is known explicitly. Given the configuration of OBFNs and all nominal parameters required, it was verified in simulation that the optimization algorithm can be used to tune optical beamforming networks for any group delays.

Keywords: *optical beamforming network, optical ring resonator, phased array antenna, non-linear optimization*

1. Introduction

In recent years, an increasing demand for fast information and data exchange on intercontinental flights has motivated the development of airplane-sattelite communication. The planes should focus the transmission or reception beams towards the satellite. Ordinary omni-directional antennas and mechanical-steered antennas are not preferable because of low gain [1], high maintenance cost, large dimension and increased drag forces [2][3]. As an alternative solution, Phased Array Antenna (PAA), an array of a very small and flat antenna elements [4] is used. PAA is fast, has low maintenance cost, and are able to reduce drag forces [5] making it a feasible solution for planes-sattelite communication.

The main element of a PAA system is a beamforming network where the delay values of the received signals are tuned to match the desired delays [6]. The beamforming network, illustrated in Figure 1, ensures the time-delayed signal adds up in phase [7]. In this paper, the desired signal and its time-delayed version are known explicitly. Cascaded Optical Ring Ressonator-based Optical Beamforming Networks is used in this paper [8][9][10].

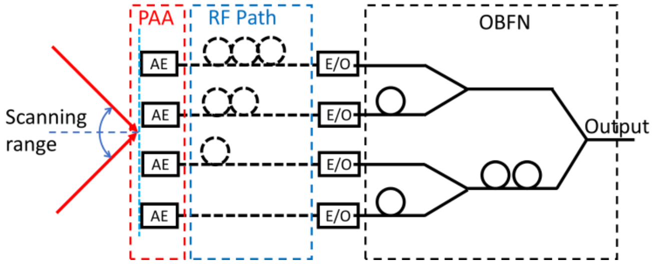

Figure 1. A phased array antenna (PAA) system, antenna elements (AEs) and optical beamforming network (OBFN)[2].

1.1. *Mathematical Model of a Cascaded Optical Ring Resonators*

A. *Optical Ring Resonator's Frequency Response*

Optical ring resonators are used as the delay elements in the optical beamforming networks. The behavior of the optical ring resonator as a delay element can be described by its frequency response. This frequency response relates the magnitude and phase of the input to the output signals. In order to determine the frequency response of an optical ring resonator, we can consider a simple one-input one-output single-stage ORR is illustrated in Figure 2 (Left) which consists of a ring-shaped and a straight waveguide. The parameter κ is the power coupling coefficient ranged between 0 and 1, L_R is the round-trip length of the ring-shaped waveguide, T is the round-trip period, and ϕ is the extra phase-shift due to heater on the top of the ring.

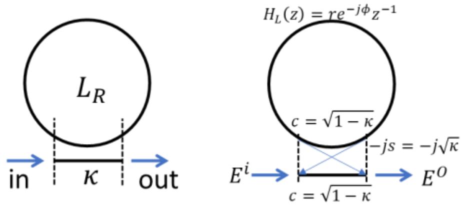

Figure 2. (Left) Structure of a 1×1 single stage ORR. (Right) z-transform schematic of ORR [2].

The Z-transform of an ORR is illustrated in Figure 2 (Right). Let the signal at the right and left side of the ring be E^r and E^l respectively, then one can derive the following relations:

$$E^r = -j\sqrt{\kappa}E^i + \sqrt{1-\kappa}E^l \quad (1)$$

$$E^l = \frac{-j\sqrt{\kappa}\, r\, z^{-1} e^{-j\phi} E^i}{1 - r\sqrt{1-\kappa}\, z^{-1} e^{-j\phi}} \quad (2)$$

$$E^o = \frac{\sqrt{1-\kappa} - r z^{-1} e^{-j\phi}}{1 - r\sqrt{1-\kappa} z^{-1} e^{-j\phi}} E^i \quad (3)$$

Where r defines the power loss. If the system is lossless then $r = 1$. Since the frequency response is defined as $H(f) = E^o/E^i$, and substituting $z^{-1} = e^{-2\pi jfT}$ with T being the round-trip period, we obtain the equation for frequency response of an ORR:

$$H(f) = \frac{\sqrt{1-\kappa} - re^{-j(2\pi fT-\phi)}}{1-r\sqrt{1-\kappa}e^{-j(2\pi fT-\phi)}} . \quad (4)$$

Equation (4) is similar to the Equation (2.18) in (2) and Equation (2.52) in (13).

B. *Optical Ring Resonator's Magnitude and Phase Response*

As has been mentioned earlier, the frequency response relates the magnitude and phase of the input to the output signals. Therefore, from the frequency response of an ORR mentioned in Equation (4), one can derive its magnitude response as follows:

$$H(f) = \frac{\sqrt{1-\kappa} - r(coscos\,(-2\pi fT-\phi) + jsinsin\,(-2\pi fT-\phi)\,)}{1-r\sqrt{1-\kappa}(coscos\,(-2\pi fT-\phi) + jsinsin\,(-2\pi fT-\phi)\,)} \quad (5)$$

$$H(f) = \frac{\sqrt{1-\kappa} - rcoscos\,(-2\pi fT-\phi) - r\,jsinsin\,(-2\pi fT-\phi)}{1-\sqrt{1-\kappa}r\,coscos\,(-2\pi fT-\phi) - j\sqrt{1-\kappa}rsinsin\,(-2\pi fT-\phi)}$$

$$|H(f)|^2 = \frac{(\sqrt{1-\kappa} - rcoscos\,(-2\pi fT-\phi)\,)^2 + (rsinsin\,(-2\pi fT-\phi)\,)^2}{(1-\sqrt{1-\kappa}rcoscos\,(-2\pi fT-\phi)\,)^2 + (\sqrt{1-\kappa}r\,sinsin\,(-2\pi fT-\phi)\,)^2},$$

$$|H(f)|^2 = \frac{(1-\kappa) + r^2 - 2\sqrt{1-\kappa}\,r\,cos(2\pi fT+\phi)}{1+(1-\kappa)r^2 - 2\sqrt{1-\kappa}rcoscos\,(2\pi fT+\phi)}, \quad (6)$$

Therefore, the magnitude response is

$$|H(f)| = \sqrt{\frac{(1-\kappa) + r^2 - 2\sqrt{1-\kappa}\,r\,cos(2\pi fT+\phi)}{1+(1-\kappa)r^2 - 2\sqrt{1-\kappa}rcoscos\,(2\pi fT+\phi)}}. \quad (7)$$

From Equation (4), we can derive the phase response of the optical ring resonator as well. The phase response $\varphi(f)$ is defined as follows:

$$\varphi(f) = arctan\ arctan\ \left(\frac{I\{H(f)\}}{R\{H(f)\}}\right),$$

$$\varphi(f) = arctan\ arctan\ \left(\frac{I\{\sqrt{1-\kappa} - re^{-j(2\pi fT-\phi)}\}}{R\{\sqrt{1-\kappa} - re^{-j(2\pi fT-\phi)}\}}\right) - arctan\ arctan\ \left(\frac{I\{1-r\sqrt{1-\kappa}e^{-j(2\pi fT-\phi)}\}}{R\{1-r\sqrt{1-\kappa}e^{-j(2\pi fT-\phi)}\}}\right),$$

$$\varphi(f) = arctan\ arctan\ \left(\frac{(-2\pi fT-\phi)}{\sqrt{1-\kappa} - rcoscos\,(-2\pi fT-\phi)}\right) -$$

$$arctan\ arctan\ \left(\frac{\sqrt{1-\kappa}r\,sinsin\,(-2\pi fT-\phi)}{1-\sqrt{1-\kappa}rcoscos\,(-2\pi fT-\phi)}\right), \quad (8)$$

Equation (7) and (8) shows the magnitude and phase response of a single optical ring resonator respectively. Knowing the equation for magnitude and phase response, we can now derive the equation for a group delay response.

C. *Optical Ring Resonator's Group Delay Response*

Group delay response is defined as the negative derivative of the phase response with respect to the frequency [11]. In order to understand how we can derive the group delay response, we can consider the following simple system:

$$y(t) = \int_{-\infty}^{\infty} x(u)h(t-u)\,du \Leftrightarrow Y(s) = H(s)X(s)$$

Let the input to the system be $x(t) = e^{i\omega t}$, the the output will be

$$y(t) = |H(i\omega)|e^{i(\omega t+\varphi(\omega))},$$

where $\varphi(\omega) = arg\ arg\ \{H(i\omega)\}$. The group delay is then defined by

$$\tau(f) = -\frac{1}{2\pi}\frac{\partial\varphi}{\partial f}. \quad (9)$$

Since the derivative of $arctan\ arctan\ \left(\frac{A(f)}{B(f)}\right)$ is given by

$$\left[arctan\ arctan\ \left(\frac{A(f)}{B(f)}\right)\right]' = \frac{B(f)A'(f) - A(f)B'(f)}{(A(f))^2 + (B(f))^2},$$

by letting $c = \sqrt{1-\kappa}$ and $z = 2\pi f T + \phi$, we can find the derivative of the phase response with respect to the frequency which will solve the Equation (9) as follows:

$$\tau(f) = -\frac{1}{2\pi}\frac{\partial\varphi}{\partial f} = -\frac{1}{2\pi}\frac{\partial\varphi}{\partial z}\frac{\partial z}{\partial x} = -T\frac{\partial\varphi}{\partial z}$$

$$\tau(f) = -T\frac{\partial}{\partial z}\left[arctan\ arctan\ \left(\frac{r sin sin\ (z)}{c - r cos cos\ (z)}\right) - arctan\ arctan\ \left(\frac{rc sin sin\ (z)}{1 - rc cos cos\ (z)}\right)\right]$$

$$\tau(f) = -T\left(\frac{-r^2 + rc cos cos\ (z)}{r^2 + c^2 - 2rc cos cos\ (z)} - \frac{-r^2c^2 + rc cos cos\ (z)}{r^2c^2 + 1 - 2rc cos cos\ (z)}\right)$$

$$\tau(f) = T\left(\frac{r^2 - rc cos cos\ (z)}{r^2 + c^2 - 2rc cos cos\ (z)} + \frac{-r^2c^2 + rc cos cos\ (z)}{r^2c^2 + 1 - 2rc cos cos\ (z)}\right) \quad (10)$$

From equation (10), we understand that the group delay response depends on the parameters κ, ϕ and T. We can observe the change of group delay response to different value of κ in Figure 3 (a). It shows when the group delay increases, i.e. κ decreases, the width of the delay curve decreases. This is due to the fact that the area under the group delay curve represents the phase shift of the ORR, which is constant 2π for one free spectral range (FSR) [11]. This observation reveals the tradeoff between the delay value and the bandwidth. A single ORR will not be able to cover large bandwidth and high desired delays at the same time. We can use cascade of multiple ORRs to solve this problem, as illustrated in Figure 3(b).

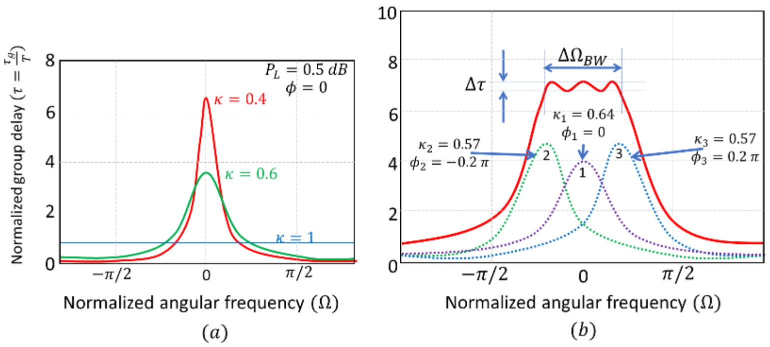

Figure 3. Group delay response of (a) single ORR, (b) cascade of multiple ORRs.

D. *Cascaded Optical Ring Resonators*

The frequency response of N-stage cascade ORRs in normalized angular frequency is defined by the product of those of the individual single-stage ORRs:

$$H_{total}(f) = \prod_{i=1}^{N} H_i(f), \quad (11)$$

where $H_i(f)$ is the frequency response of a single ORR in the stage i. The magnitude, phase and group delay response are respectively defined by:

$$|H_{total}(f)| = \prod_{i=1}^{N} |H_i(f)|, \quad (12)$$

$$\varphi_{total}(f) = \sum_{i=1}^{N} \varphi_i(f), \quad (13)$$

$$\tau_{total}(f) = \sum_{i=1}^{N} \tau_i(f). \quad (14)$$

After knowing the magnitude, phase and group delay response given in Equation (12-14), now we can derive the cost function based on the group delay of a cascaded optical ring resonator, and then determine the optimization algorithm to solve the tuning problem. The derivation of the cost function and optimization algorithm will be presented in the next section.

2. METHODE

A. Cost Function: Group Delay

The optimum parameters of the cascaded optical ring resonators are found by minimizing a certain cost function. The cost function what we used in this paper is the group delay function. The optimum parameters of the cascaded optical ring resonators will minimize the value of the group delay. If the group delay function is minimum, the phase difference between received signals will be minimum as well, which in consequence will create maximum output signal power.

The group delay cost function is based on the delay spectrum formula for one optical ring resonator mentioned in Equation (10). For a simplicity, consider the lossless optical ring resonator system, therefore r=1. By substituting Equation (10) with r=1, we got the following equation:

$$\tau(f) = \frac{\kappa T}{2-\kappa-2\sqrt{1-\kappa}coscos\,(2\pi f T+\phi)}, \tag{15}$$

where $\tau(f)$ is the group delay for a specific frequency, κ is the coupling coefficient, ϕ is an extra phase shift, and T is the round-trip time.

For the case of cascaded optical ring resonators, the index i is added to specify the group delay for different optical ring resonators. For each path of the optical beamforming network, the delay responses should be summed to obtain the total delay response (τ_{tot}) for one path. Therefore, the formula to obtain the total delay response for a single path of optical beamforming network is

$$\tau_{tot}(f) = \sum_{i=1}^{N} \frac{\kappa_i T}{2-\kappa_i-2\sqrt{1-\kappa_i}coscos\,(2\pi f T+\phi_i)}. \tag{16}$$

It is important to note that the value of the delay we consider is only in the part of the spectrum where the modulated signals is located. It means that the delay we need to consider in the cost function is only the delay over some specific range of frequency. Therefore, the comparison of the group delay and the desired delay should be carried out in a certain frequency band, defined by a starting frequency f_{min} and end frequency f_{max}.

In this paper, it is assumed that the frequency from f_{min} to f_{max} is given. The total group delay cost function is obtained by summing the total group delay for all path and for all frequency inside the frequency range and compare it to the sum of target delays. Then, the cost function we want to minimize is

$$\mu(\kappa_1,\phi_1,\kappa_2,\phi_2,\cdots) = \sum_{h=1}^{M} \sum_{k=1}^{P} \left(\tau_{tot_h}(f_k) - D_h\right)^2, \tag{17}$$

where μ is the cost function, h is the path index of OBFN, M is the number of path of the OBFN, and k is the k-th frequency in the given frequency range P.

B. Optimization Method

After the cost function has been derived, we now present the method of optimization. Non-Linear Programming (NLP) (11, 12) is the method that is proposed in this paper. NLP is used to find the minimum value of cost function subject to several constraints. The cost function used in mentioned in Equation (17).

The specification of the constraints in the optimization method is quite trivial. The value of κ must be in the range between 0 and 1, and the phase shift ϕ is limited from 0 to 2π. This limitation of ϕ is due to the fact that the phase shift of more than 2π is theoretically not meaningful.

The optimization problem can be formulized as follows:

$$\mu(\kappa_1,\phi_1,\kappa_2,\phi_2,\cdots) = \sum_{h=1}^{M} \sum_{k=1}^{P} \left(\tau_{tot_h}(f_k) - D_h\right)^2 . \tag{18}$$

$$s.t. \qquad 0 < \kappa < 1$$
$$0 < \phi < 2\pi$$

3. RESULT

A. Simulation Setup

The nominal parameters of OBFN setup simulated in this project are similar to the ones used in (16). Table I shows all parameters needed and their value. The target delays and input signals are given.

Table 1. The Nominal Parameters of the OBFN

Symbol	*Quantity*	*Value*	*Symbol*	*Quantity*	*Value*
r	Power factor	$1^{a)}$	f_c	Frequency center	107.52 Hz
$\lvert H(f)\rvert$	Magnitude of freq response	$1^{a)}$	ω	Bandwidth of interest	2 GHz
τ	Round-trip period	0.08 ns	N	Number of frequency	$100^{b)}$
λ	wavelength	1550 nm			

[a] We assume that the system is lossless, therefore the power factor $r = 1$, and magnitude of frequency response of ORR $|H(f)| = 1$.

[b] Number of frequencies should be chosen sufficiently big, i.e., $N = 100$ for the sake of computation accuracy.

The results will show the group delay response graph and test error. This test error refers to the difference between the desired output and the actual output of the neural network for given training examples, which is stated in Equation (17). It is also necessary to compute normalized squared group delay error which is defined by

$$E = \frac{1}{2}\sum_{m=1}^{M} \sum_{n=1}^{N} \frac{\|D_{des,m}(f_n) - D_{act,m}(f_n)\|^2}{\|D_{des,m}(f_n)\|^2} \tag{19}$$

where M specifies the number of AE, $D_{des,m}$ and $D_{act,m}$ specify the desired and actual delay response of the m-th path respectively. The normalized squared group delay error is essential since it gives the comparison how big the error is compared to the desired delay response.

B. Simulation Result

Figure 4 shows group delay response and test error of simulation result of a 4×1 OBFN with desired delay (0 0.1 0.2 0.3) ns. Table 2 shows the optimum value of κ^* and ϕ^*, and the initial and final normalized squared group delay error (E).

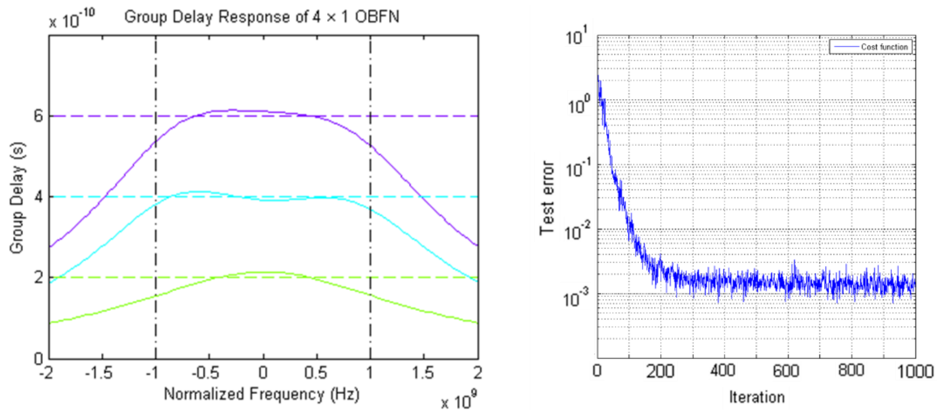

Figure 4. Group delay and test error of OBFN with desired delay (0 0.1 0.2 0.3) ns.

Table 2. Comparison between initial (κ_0 and ϕ_0) - optimum (κ^* and ϕ^*) ORR parameter values using nominal parameters mentioned in Table 1.

κ_0		ORR 1	ORR 2	ORR 3	ORR 4	Final E
(0.9 0.9 0.9 0.9)	κ^*	0.9870	0.9873	0.9803	0.9933	2.0235×10^{-4}
	ϕ^*	0.0018	5.9191	0.3713	0.0027	
(0.7 0.7 0.7 0.7)	κ^*	0.9867	0.9870	0.9798	0.9867	1.9079×10^{-4}
	ϕ^*	0.0024	5.8783	0.4059	0.0012	
(0.5 0.5 0.5 0.5)	κ^*	0.9867	0.9861	0.9797	0.9868	1.8535×10^{-4}
	ϕ^*	0.0032	5.8729	0.4199	0.0033	
(0.3 0.3 0.3 0.3)	κ^*	0.9867	0.0326	0.0295	0.5986	2.2898
	ϕ^*	0.0032	5.8729	0.4199	0.0033	

Table 3 shows the simulation result of 4×1 OBFN for different initial guess of κ with desired delay (0 0.1 0.2 0.3) ns. We can observe that if we set the initial guess of parameters incorrectly, the optimization process may be stuck in the local optima, as we can see when we set the initial parameter $\kappa_0 = (0.3\ 0.3\ 0.3\ 0.3)$. When the optimization process is stuck in the local optima, the final error will be very big, and therefore the output power will not be maximum. Obviously, this is not desirable. In order to avoid local optima, we should choose the initial guess of the parameters carefully.

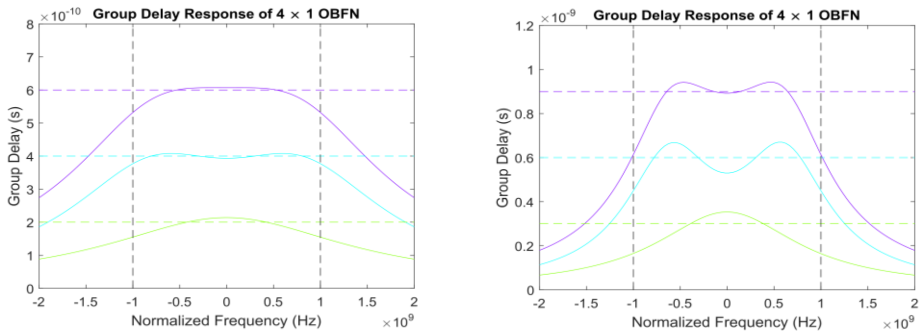

Figure 5. Simulation result of a 4×1 OBFN with desired delay (left) (0 0.2 0.4 0.6) ns and (right) (0 0.3 0.6 0.9) ns.

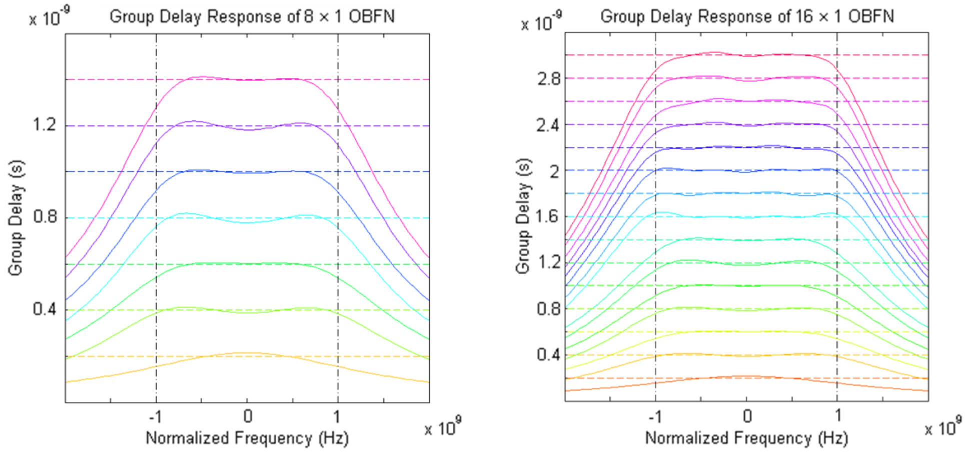

Figure 6. Simulation result of (left) 8×1 OBFN, and (right) 16×1 OBFN.

Figure 5 shows the group delay responses of 4×1 OBFN, with desired delay $\delta\times[0\ 0.1\ 0.2\ 0.3]$ ns where we use $\delta=2$ and $\delta=3$ respectively. We can observe that as the desired group delay increases, the ripple of the delay response will increase as well. Figure 6 shows the group delay responses of 8×1 and 16×1 OBFN setups. We can observe that the non-linear programming indeed can be used to tune even larger OBFN setups.

4. CONCLUSION

Cascaded optical ring resonator-based optical beamforming networks can be used to control Phased Array Antennas (PAAs) such that planes can communicate to satellites by directing the transmission/reception beam towards sattelites. Non-Linear Programming (NLP) optimization method is proposed to control the optical beamforming networks such that the optimum parameters of the optical ring resonator are obtained. These optimum parameters will result in maximum output signal power. Given a certain OBFN structure, a Non-Linear Programming (NLP) optimization works well to find the optimum parameters for 4×1, 8×1, and even 16×1 OBFN for any given desired delays. However, it is important to note that the algorithm may be stuck in local optima if the initial guess of the parameters is chosen poorly. Future development to deal with these local optima problem is needed.

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